EE 204 Signals and Systems Laboratory 9

I. PREPARATION

1) Background Information:

$x(t)$ is an aperiodic signal.

**** Fourier transform of** $x(t) \rightarrow \boxed{x(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt}$ (1)

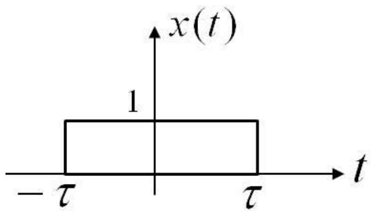
**** Inverse Fourier transform** $\rightarrow \boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(\omega)e^{j\omega t}\,d\omega}$ (2)

Duality Property:

$$f(t) \xleftrightarrow{FT} g(\omega)$$

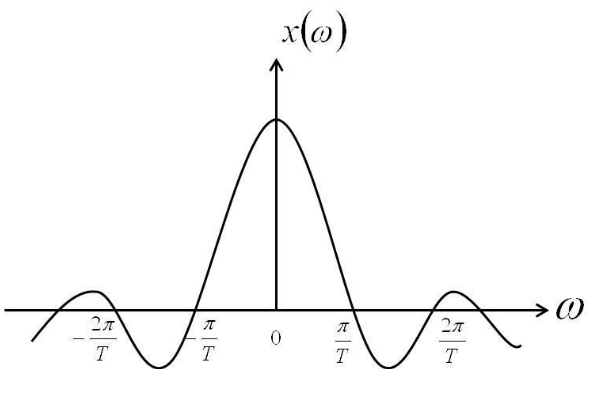
$$g(t) \xleftrightarrow{FT} 2\pi * f(-\omega)$$

Consider the $x(t)$ signal given below;



Let apply Equation (1) to this $x(t)$ signal:

$$x(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt = \int_{-\tau}^{\tau} 1e^{-j\omega t}\,dt = \frac{2}{\omega}\mathbf{sin(}\omega\tau)$$



$x(\omega)$ is the Fourier transform of $x(t)$ and it is plotted above. Let assume that $\tau = 1$, so the final expression of $x(\omega)$ becomes a sinc function:

$$x(\omega) = \frac{2\mathbf{sin(}\omega)}{\omega} = 2\mathbf{sinc(}\omega\mathbf{)}$$

Therefore, the Fourier transform of a rectangle function is a sinc function.

II. EXPERIMENTAL WORK

1) Plot $x(t)$ and $x(\omega)$ signals using Matlab. Observe what happens when $\tau \to \infty$ and $\tau \to 0$. Plot graphs of $x(t)$ and $x(\omega)$ for $\tau \to \infty$ and $\tau \to 0$ cases.

2) Using duality find Fourier transform of $x(t) = \frac{1}{\pi t}\sin(t\tau)$. Plot $x(t)$ and $x(\omega)$ for large and small values of τ.

Hint:

Since $x(t) = \frac{1}{\pi t}\sin(t\tau)$, by using duality $x(\omega)$ can be found as;

$$x(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(t)e^{j\omega t}\,dt$$