

System: $y(t) = x(t) * h(t)$

For continuous time $f(t^-) = f(t^+)$

For discrete time $f[n]$ may not be equal to $f[n+1]$ or $f[n-1]$

Energy & Power:

$$E_c = \int_{t_1}^{t_2} |x(t)|^2 dt \longrightarrow P_c = \frac{E_c}{t_2 - t_1}$$

$$E_d = \sum_{n=-N}^N |x[n]|^2 \longrightarrow P_d = \frac{E_d}{2N + 1}$$

$$E_\infty = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt \quad P_\infty = \frac{E_\infty}{2T}$$

$$|x[n]| = \sqrt{x[n] x^*[n]} \quad P_\infty = \frac{E_\infty}{2N + 1}$$

Periodicity of Signals:

$x(t) = x(t + T)$ for continuous time

$x[n] = x[n + N]$ for discrete time

$$e^{j2\pi m} = 1$$

Properties of Impulse Function:

$$1) \delta(t - t_0) f(t) = \delta(t - t_0) f(t_0) = f(t_0)$$

$$2) \delta(t - t_0) f(t - t_1) = \delta(t - t_0) f(t_0 - t_1)$$

$$3) \int_{-\infty}^{\infty} \delta(t) dt = 1 \text{ OR } \int_{-\infty}^{\infty} \delta(t - t_0) dt = 1$$

$$4) \int_{-\infty}^{\infty} \delta(t - t_0) f(t) dt = f(t_0),$$

$$\int_{-\infty}^{\infty} \delta(t - t_0) f(t - t_1) dt = f(t_0 - t_1)$$

Ramp Function:

$$u(t) = \frac{dr(t)}{dt}, r(t) = \int_{-\infty}^t u(t) dt$$

$$\delta(t) = \frac{du(t)}{dt}, u(t) = \int_0^{\infty} \delta(t) dt$$

$$\delta(t) = \frac{d^2 r(t)}{dt^2} \quad \delta(at) = \frac{1}{|a|} \delta(t)$$

$$r(t) \xleftrightarrow[\text{integration}]{\text{derivative}} u(t) \xleftrightarrow[\text{integration}]{\text{derivative}} \delta(t)$$

Even and Odd Signals:

Even $\rightarrow x(t) = x(-t)$, Odd $\rightarrow x(t) = -x(t)$

Exponential and Sinusoidal Signals:

$$e^{j\omega_0 t} = \cos(\omega_0 t) + j \sin(\omega_0 t) \quad \omega_0 = 2\pi f$$

Discrete Time Signal, Base Functions:

$$u[n] = \sum_{k=0}^{\infty} \delta[n - k]$$

$$r[n] = \sum_{k=0}^{\infty} u[n - k] = \sum_{k=0}^{\infty} k \delta[n - k]$$

Linearity:

$$ax_1(t) + bx_2(t) = ay_1(t) + by_2(t)$$

Linear Time Invariant Systems:

The Convolution Sum:

$$g[n] = \sum_{k=-\infty}^{\infty} x[k] \times h[n - k]$$

$$y[n] = \sum_{k=0}^n \alpha^k = \frac{1 - \alpha^{n+1}}{1 - \alpha}$$

Properties of Convolution:

Commutative Property:

$$x[n] * h[n] = h[n] * x[n]$$

Distributive Property:

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$$

Associative Property:

$$x(t) * [h_1(t) * h_2(t)] = (x(t) * h_1(t)) * h_2(t)$$

Invertability of LTI Systems:

$$x(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau) d\tau$$

Stability for LTI Systems:

LTI system is stable if and only if

$$\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$$

Unit Step Response:

For discrete time

$$s[n] = u[n] * h[n]$$

$$s[n] = \sum_{k=-\infty}^{\infty} h[k]u[n-k] \quad n \geq k$$

Fourier Analysis for Continuous Time Signals and Systems :

$$x(t) = A[0] + \sum_{k=1}^{\infty} A[k] \cos\left(\frac{2\pi}{T} kt\right) + \sum_{k=1}^{\infty} B[k] \sin\left(\frac{2\pi}{T} kt\right)$$

$$A[0] = \frac{1}{T} \int_T x(t) dt$$

$$A[k] = \frac{2}{T} \int_T x(t) \cos\left(k \frac{2\pi}{T} t\right) dt$$

$$B[k] = \frac{2}{T} \int_T x(t) \sin\left(k \frac{2\pi}{T} t\right) dt$$

Fourier series representation

$$x(t) = \sum_{k=-\infty}^{\infty} x[k] e^{jk \frac{2\pi}{T} t}$$

$$x[k] = \frac{1}{T} \int_T x(t) e^{-jk \frac{2\pi}{T} t} dt$$

$$A[k] = (x[k] + x^*[k]) / 2 \quad k \neq 0$$

$$B[k] = (x[k] - x^*[k]) / j$$

$$A[k] = x[0] \quad \text{OR} \quad x[k] = \frac{1}{2} (A[k] - jB[k])$$

$$A[0] = x[0]$$

$$y(t) = \frac{d^n x(t)}{dt^n} \quad \text{then} \quad y[k] = (j\omega_0 k)^n \times x[k]$$

$$y(t) = \frac{d^n x(t)}{dt^n} \quad x[k] = \left(\frac{1}{jk\omega_0} \right)^n y[k]$$

$$\sin \theta = \frac{1}{2j} (e^{j\theta} - e^{-j\theta})$$

$$\cos \theta = \frac{1}{2} (e^{j\theta} + e^{-j\theta})$$

Properties of Fourier Series Coefficients:

- | <u>Signal</u> | <u>FSC</u> |
|---------------------------------------|---|
| 1. $Ax(t) + By(t)$ | $\xleftrightarrow{FSC} Ax[k] + By[k]$ |
| 2. $x(t - t_0)$ | $\xleftrightarrow{FSC} x[k] e^{-jk \frac{2\pi}{T} t_0}$ |
| 3. $e^{jm \frac{2\pi}{T} t} x(t)$ | $\xleftrightarrow{FSC} x[k - m]$ |
| 4. $x^*(t)$ | $\xleftrightarrow{FSC} x^*[-k]$ |
| 5. $x(-t)$ | $\xleftrightarrow{FSC} x[-k]$ |
| 6. $\int_T x(\tau) y(t - \tau) d\tau$ | $\xleftrightarrow{FSC} Tx[k] y[k]$ |
| 7. | |
| $x(t) y(t)$ | $\xleftrightarrow{FSC} x[k] * y[k] = \sum_{l=-\infty}^{\infty} x[l] y[k - l]$ |
| 8. $\frac{d^n x[n]}{dt^n}$ | $\xleftrightarrow{FSC} (jk\omega_0)^n x[k]$ |
| 9. $\int_{-\infty}^t x(t) dt$ | $\xleftrightarrow{FSC} \frac{1}{jk\omega_0} x[k]$ |
| 10. $\int_T x(t) ^2 dt$ | $\xleftrightarrow{FSC} T \sum_{k=-\infty}^{\infty} x[k] ^2$ |

Representation of a periodic Signals:

The continuous time fourier transform:

$$\widetilde{x(t)} = \sum_{k=-\infty}^{\infty} x(t - kT)$$

$x(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$ (Fourier transform of a Periodic Signal x(t))

$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(\omega) e^{j\omega t} d\omega$ (Inverse Fourier Transform of a Periodic Signal)

Convergence of Fourier Transform:

$$e^{j\theta} = \cos \theta + j \sin \theta$$

$$|e^{j\theta}| = \sqrt{\cos^2 \theta + \sin^2 \theta} = 1$$

$$\int_{-\infty}^{\infty} |x(t)| dt < \infty$$

Relation between FSC of Periodic Signal:

$$\widetilde{x[k]} = \frac{1}{T} x(w) \Big|_{w=kw_0}, \quad w_0 = \frac{2\pi}{T}$$

$$e^{jw_0 t} \xrightarrow{F.T} 2\pi \delta(w - w_0)$$

Properties of Fourier Transform:

$$|x(w)| = \sqrt{a^2(w) + b^2(w)}$$

$$\tan^{-1} = \frac{b(w)}{a(w)}$$

$$\frac{d^n x(t)}{dt^n} \xrightarrow{FT} (jw)^n x(w)$$

Fourier Analysis for Discrete Time Signals and Systems:

$$x[n] = \sum_{-\infty}^{N-1} x[k] e^{jk \frac{2\pi}{N} n}$$

$$x[k] = \frac{1}{N} \sum_{n, N} x[n] e^{-jk \frac{2\pi}{N} n}$$

$$\sum_{n=0}^{N-1} r^n = 1 + r + r^2 + r^3 + \dots = \frac{1-r^N}{1-r} \quad \text{if } r \neq 1$$

$$\sum_{n=0}^{N-1} r^n = 1 + r + r^2 + r^3 + \dots = N \quad \text{if } r = 1$$

Properties of Discrete Fourier Series

Coefficients:

$$1. \alpha x[n] + \beta y[n] \xrightarrow{DFSC} \alpha x[k] + \beta y[k]$$

$$2. x[n - n_0] \xrightarrow{DFSC} e^{-jk \frac{2\pi}{N} n_0} x[k]$$

$$3. e^{jm \frac{2\pi}{N} n} x[n] \xrightarrow{DFSC} x[k - m]$$

$$4. x^*[n] \xrightarrow{DFSC} x^*[k]$$

$$5. x[-n] \xrightarrow{DFSC} x[-k]$$

$$6. \sum_{m, n} x[m] y[n - m] \xrightarrow{DFSC} Nx[k] y[k]$$

$$7. x[n] y[n] \xrightarrow{DFSC} \sum_{l, N} x[l] y[k - l]$$

$$8. x[n] - x[n - l] \xrightarrow{DFSC} (1 - e^{-jk \frac{2\pi}{N} l}) x[k]$$

9.

$$y[n] = \begin{cases} x[n/m] & \text{if } n \text{ is multiple of } m \\ 0 & \text{else} \end{cases} \xrightarrow{DFSC} y[k] = \frac{1}{m} x[k]$$

$$10. \sum_{k=-\infty}^n x[k] \xrightarrow{DFSC} \frac{x[k]}{1 - e^{-jk \frac{2\pi}{N}}}$$

The Laplace Transform:

$$x(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt = FT(x(t) e^{-\theta t})$$

Properties of Laplace Transform:

$$x_1(t) \xrightarrow{L} x_1(s) \quad ROC = R_1$$

$$x_2(t) \xrightarrow{L} x_2(s) \quad ROC = R_2$$

$$x(t) \xrightarrow{L} x(s) \quad ROC = R$$

1) Linearity

$$ax_1(t) + bx_2(t) \xrightarrow{L} ax_1(s) + bx_2(s)$$

$$ROC \quad R_1 \cap R_2$$

2) Time shifting $x(t) \xrightarrow{L} x(s) \quad ROC = R$

$$x(t - t_0) \xrightarrow{L} e^{-st_0} x(s) \quad \text{with } ROC = R$$

3) Shifting in the S-Domain

$$x(t) \xrightarrow{L} x(s), \quad ROC = R$$

$$e^{s_0 t} x(t) \xrightarrow{L} x(s - s_0)$$

$$ROC = R + \text{Re}\{s_0\}$$

4) Time Scaling

$$x(at) \xrightarrow{L} \frac{1}{|a|} x\left(\frac{s}{a}\right), \quad ROC = \frac{R}{a}$$

5) Convolution Property

$$x_1(t) * x_2(t) \xrightarrow{L} x_1(s) x_2(s)$$

$$ROC = R_1 \cap R_2$$

6) Differentiation in Time Domain

$$\frac{dx(t)}{dt} \xrightarrow{L} sx(s) \quad ROC > R$$

7) Differentiation in s-domain

$$-tx(t) \xrightarrow{L} \frac{dx(s)}{ds} \quad ROC = R$$

8) Integration in the time domain

$$\int_{-\infty}^t x(\tau) d\tau \xrightarrow{L} \frac{1}{s} x(s)$$

$$ROC > R \cap \{\text{Re}(s) > 0\}$$

9) The initial and final value theorems

$$x(0^+) = \lim_{s \rightarrow \infty} sx(s) \quad \lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sx(s)$$

The Unilateral Laplace Transform:

$$\frac{dx(t)}{dt} \xrightarrow{L} sx(s) - x(0)$$

Second derivative of x(t):

$$L\{x''(t)\} = s^2 x(s) - sx(0) - x'(0)$$

The Z-Transform:

$$x(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}, \quad x(z) = FT\{x[n] r^{-n}\}$$

Note:

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}, \quad |r| < 1$$

Properties of Z-Transform

$$x_1[n] \xrightarrow{z} x_1[z] \quad ROC_1$$

$$x_2[n] \xrightarrow{z} x_2[z] \quad ROC_2$$

1) Linearity

$$\alpha x_1[n] + \beta x_2[n] \xrightarrow{z} \alpha x_1[z] + \beta x_2[z]$$

$$ROC = ROC_1 \cap ROC_2$$

2) Time Shifting

$$x[n] \xrightarrow{z} x[z] \quad ROC = R_x$$

$$x[n - n_0] \xrightarrow{z} z^{-n_0} x[z] \quad ROC = R_x$$

3) Frequency Shifting

$$e^{jn_0} x[n] \xrightarrow{z} x(e^{-jn_0} z) \quad ROC = R_x$$

$$z_0^n x[n] \xrightarrow{z} x\left(\frac{z}{z_0}\right) \quad ROC = z_0 R_x$$

4) Time Reversal

$$x[-n] \xrightarrow{z} x\left(\frac{1}{z}\right), \quad ROC = \frac{1}{R_x}$$

5) Convolution Property

$$x_1[n] * x_2[n] \xrightarrow{z} x_1(z) x_2(z)$$

$$ROC = ROC_1 \cap ROC_2$$

6) Differentiation in the z-domain

$$nx[n] \xrightarrow{z} -\frac{z dx(z)}{dz}, \quad ROC = R_x$$

7) The initial value theorem

$$\text{If } x[n] = 0, \quad n < 0 \text{ then } x[0] = \lim_{z \rightarrow \infty} x(z)$$

Table of z-Transform Pairs

$x[n] = \mathcal{Z}^{-1}\{X(z)\} = \frac{1}{2\pi j} \oint X(z)z^{n-1}dz$		\longleftrightarrow	$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{+\infty} x[n]z^{-n}$	ROC
transform	$x[n]$	\longleftrightarrow	$X(z)$	R_x
time reversal	$x[-n]$	\longleftrightarrow	$X(\frac{1}{z})$	$\frac{1}{R_x}$
complex conjugation	$x^*[n]$	\longleftrightarrow	$X^*(z^*)$	R_x
reversed conjugation	$x^*[-n]$	\longleftrightarrow	$X^*(\frac{1}{z^*})$	$\frac{1}{R_x}$
real part	$\Re\{x[n]\}$	\longleftrightarrow	$\frac{1}{2}[X(z) + X^*(z^*)]$	R_x
imaginary part	$\Im\{x[n]\}$	\longleftrightarrow	$\frac{1}{2j}[X(z) - X^*(z^*)]$	R_x
time shifting	$x[n - n_0]$	\longleftrightarrow	$z^{-n_0}X(z)$	R_x
scaling in \mathcal{Z}	$a^n x[n]$	\longleftrightarrow	$X(\frac{z}{a})$	$ a R_x$
downsampling by N	$x[Nn], N \in \mathbb{N}_0$	\longleftrightarrow	$\frac{1}{N} \sum_{k=0}^{N-1} X(W_N^k z^{\frac{1}{N}})$ $W_N = e^{-j\frac{2\pi}{N}}$	R_x
linearity	$ax_1[n] + bx_2[n]$	\longleftrightarrow	$aX_1(z) + bX_2(z)$	$R_x \cap R_y$
time multiplication	$x_1[n]x_2[n]$	\longleftrightarrow	$\frac{1}{2\pi j} \oint X_1(u)X_2(\frac{z}{u})u^{-1}du$	$R_x \cap R_y$
frequency convolution	$x_1[n] * x_2[n]$	\longleftrightarrow	$X_1(z)X_2(t)$	$R_x \cap R_y$
delta function	$\delta[n]$	\longleftrightarrow	1	$\forall z$
shifted delta function	$\delta[n - n_0]$	\longleftrightarrow	z^{-n_0}	$\forall z$
step	$u[n]$	\longleftrightarrow	$\frac{z}{z-1}$	$ z > 1$
	$-u[-n - 1]$	\longleftrightarrow	$\frac{z}{z-1}$	$ z < 1$
ramp	$nu[n]$	\longleftrightarrow	$\frac{z}{(z-1)^2}$	$ z > 1$
	$n^2u[n]$	\longleftrightarrow	$\frac{z(z+1)}{(z-1)^3}$	$ z > 1$
	$-n^2u[-n - 1]$	\longleftrightarrow	$\frac{z(z+1)}{(z-1)^3}$	$ z < 1$
	$n^3u[n]$	\longleftrightarrow	$\frac{z(z^2+4z+1)}{(z-1)^4}$	$ z > 1$
	$-n^3u[-n - 1]$	\longleftrightarrow	$\frac{z(z^2+4z+1)}{(z-1)^4}$	$ z < 1$
	$(-1)^n$	\longleftrightarrow	$\frac{z}{z+1}$	$ z < 1$
exponential	$a^n u[n]$	\longleftrightarrow	$\frac{z}{z-a}$	$ z > a $
	$-a^n u[-n - 1]$	\longleftrightarrow	$\frac{z}{z-a}$	$ z < a $
	$a^{n-1} u[n - 1]$	\longleftrightarrow	$\frac{1}{z-a}$	$ z > a $
	$na^n u[n]$	\longleftrightarrow	$\frac{az}{(z-a)^2}$	$ z > a $
	$n^2 a^n u[n]$	\longleftrightarrow	$\frac{az(z+a)}{(z-a)^3}$	$ z > a $
	$e^{-an} u[n]$	\longleftrightarrow	$\frac{z}{z-e^{-a}}$	$ z > e^{-a} $
exp. interval	$\begin{cases} a^n & n = 0, \dots, N - 1 \\ 0 & \text{otherwise} \end{cases}$	\longleftrightarrow	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$ z > 0$
sine	$\sin(\omega_0 n) u[n]$	\longleftrightarrow	$\frac{z \sin(\omega_0)}{z^2 - 2 \cos(\omega_0)z + 1}$	$ z > 1$
cosine	$\cos(\omega_0 n) u[n]$	\longleftrightarrow	$\frac{z(z - \cos(\omega_0))}{z^2 - 2 \cos(\omega_0)z + 1}$	$ z > 1$
	$a^n \sin(\omega_0 n) u[n]$	\longleftrightarrow	$\frac{za \sin(\omega_0)}{z^2 - 2a \cos(\omega_0)z + a^2}$	$ z > a$
	$a^n \cos(\omega_0 n) u[n]$	\longleftrightarrow	$\frac{z(z - a \cos(\omega_0))}{z^2 - 2a \cos(\omega_0)z + a^2}$	$ z > a$
differentiation in \mathcal{Z}	$nx[n]$	\longleftrightarrow	$-z \frac{dX(z)}{dz}$	R_x
integration in \mathcal{Z}	$\frac{x[n]}{n}$	\longleftrightarrow	$-\int_0^z \frac{X(z)}{z} dz$	R_x
	$\frac{\prod_{i=1}^m (n-i+1)}{a^m m!} a^m u[n]$	\longleftrightarrow	$\frac{z}{(z-a)^{m+1}}$	

Note:

$$\frac{z}{z-1} = \frac{1}{1-z^{-1}}$$

Table of Laplace Transform Pairs

$f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi j} \lim_{T \rightarrow \infty} \int_{c-jT}^{c+jT} F(s)e^{st} ds$		$\xleftrightarrow{\mathcal{L}}$	$F(s) = \mathcal{L}\{f(t)\} = \int_{-\infty}^{+\infty} f(t)e^{-st} dt$	
transform	$f(t)$	$\xleftrightarrow{\mathcal{L}}$	$F(s)$	
complex conjugation	$f^*(t)$	$\xleftrightarrow{\mathcal{L}}$	$F^*(s^*)$	
time shifting	$f(t-a) \quad t \geq a > 0$	$\xleftrightarrow{\mathcal{L}}$	$a^{-as} F(s)$	
	$e^{-at} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$F(s+a)$	frequency shifting
time scaling	$f(at)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{ a } F\left(\frac{s}{a}\right)$	
linearity	$af_1(t) + bf_2(t)$	$\xleftrightarrow{\mathcal{L}}$	$aF_1(s) + bF_2(s)$	
time multiplication	$f_1(t)f_2(t)$	$\xleftrightarrow{\mathcal{L}}$	$F_1(s) * F_2(s)$	frequency convolution
time convolution	$f_1(t) * f_2(t)$	$\xleftrightarrow{\mathcal{L}}$	$F_1(s)F_2(s)$	frequency product
delta function	$\delta(t)$	$\xleftrightarrow{\mathcal{L}}$	1	
shifted delta function	$\delta(t-a)$	$\xleftrightarrow{\mathcal{L}}$	e^{-as}	exponential decay
unit step	$u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}$	
ramp	$tu(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s^2}$	
parabola	$t^2 u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2}{s^3}$	
n -th power	t^n	$\xleftrightarrow{\mathcal{L}}$	$\frac{n!}{s^{n+1}}$	
exponential decay	e^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s+a}$	
two-sided exponential decay	$e^{-a t }$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2a}{a^2 - s^2}$	
	te^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{(s+a)^2}$	
	$(1-at)e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{(s+a)^2}$	
exponential approach	$1 - e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{a}{s(s+a)}$	
sine	$\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2 + \omega^2}$	
cosine	$\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2 + \omega^2}$	
hyperbolic sine	$\sinh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2 - \omega^2}$	
hyperbolic cosine	$\cosh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2 - \omega^2}$	
exponentially decaying sine	$e^{-at} \sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{(s+a)^2 + \omega^2}$	
exponentially decaying cosine	$e^{-at} \cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s+a}{(s+a)^2 + \omega^2}$	
frequency differentiation	$tf(t)$	$\xleftrightarrow{\mathcal{L}}$	$-F'(s)$	
frequency n -th differentiation	$t^n f(t)$	$\xleftrightarrow{\mathcal{L}}$	$(-1)^n F^{(n)}(s)$	
time differentiation	$f'(t) = \frac{d}{dt} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$sF(s) - f(0)$	
time 2nd differentiation	$f''(t) = \frac{d^2}{dt^2} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^2 F(s) - sf(0) - f'(0)$	
time n -th differentiation	$f^{(n)}(t) = \frac{d^n}{dt^n} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$	
time integration	$\int_0^t f(\tau) d\tau = (u * f)(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s} F(s)$	
frequency integration	$\frac{1}{t} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$\int_s^\infty F(u) du$	
time inverse	$f^{-1}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s) - f^{-1}}{s}$	
time differentiation	$f^{-n}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)}{s^n} + \frac{f^{-1}(0)}{s^n} + \frac{f^{-2}(0)}{s^{n-1}} + \dots + \frac{f^{-n}(0)}{s}$	